Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2014**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

1 Two small smooth spheres A and B have equal radii and have masses m and km respectively. They are moving in a straight line in the same direction on a smooth horizontal table. The speed of A is u and the speed of B is $\frac{2}{3}u$. Sphere A collides directly with sphere B. The coefficient of restitution between the spheres is $\frac{4}{5}$.

(i) Show that the speed of A after the collision is $\dfrac{u(2k+5)}{5(k+1)}$. [5]

(ii) Given that the magnitude of the impulse experienced by B during the collision is $\frac{2}{5}mu$, find the value of k. [2]

2 A particle P of mass m kg moves on an arc of a circle with centre O and radius a metres. At time $t = 0$ the particle is at the point A. At time t seconds, angle $POA = \sin^2 2t$. Show that the radial component of the acceleration of P at time t seconds has magnitude $(4a \sin^2 4t)\ \text{m s}^{-2}$. [2]

Find

(i) the value of t when the transverse component of the acceleration of P is first equal to zero, [2]

(ii) the magnitude of the resultant force acting on P when $t = \frac{1}{12}\pi$. [2]

3 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. When the particle is hanging in equilibrium, with the string vertical, it is given a horizontal impulse of magnitude J. When the string makes an angle θ with the downward vertical at O, the tension is T.

(i) Show that $T = \dfrac{J^2}{ma} - mg(2 - 3\cos\theta)$. [5]

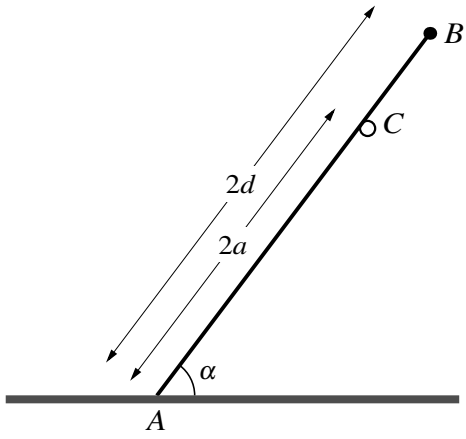
(ii) It is given that $J = m\sqrt{(kga)}$, where k is a positive constant. Justifying your answers, describe the motion of P in each of the following cases:

(a) $k = 1$,

(b) $k = 6$.

[4]

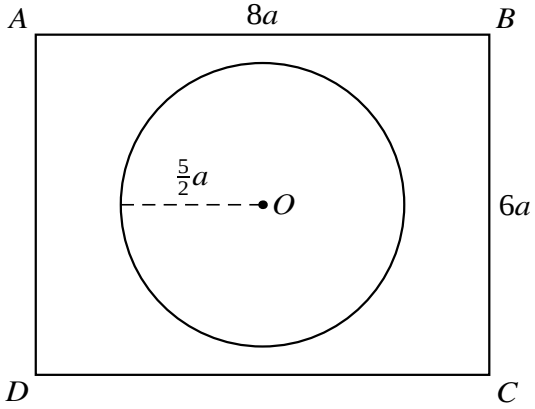
4



A uniform rod AB has mass m and length $2d$. The rod rests in equilibrium on a smooth peg C, with the end A resting on a rough horizontal plane. The distance AC is $2a$ and the angle between AB and the horizontal is α, where $\cos\alpha = \frac{3}{5}$. A particle of mass $\frac{1}{2}m$ is attached to the rod at B (see diagram). Find the normal reaction at A and deduce that $d < \frac{25}{6}a$. [6]

The coefficient of friction between the rod and the plane is μ. Show that $\mu \geqslant \dfrac{8d}{25a - 6d}$. [4]

5



A uniform rectangular lamina $ABCD$, in which $AB = 8a$ and $BC = 6a$, has mass M. A uniform circular lamina of radius $\frac{5}{2}a$ has mass $\frac{1}{3}M$. The two laminas are fixed together in the same plane with their centres coinciding at the point O (see diagram). A particle P of mass $\frac{1}{2}M$ is attached at C. The system is free to rotate about a fixed smooth horizontal axis through A and perpendicular to the plane $ABCD$. Show that the moment of inertia of the system about this axis is $\frac{2225}{24}Ma^2$. [8]

The system is released from rest with AC horizontal and D below AC. Find, in the form $k\sqrt{\left(\frac{g}{a}\right)}$, the greatest angular speed in the subsequent motion, giving the value of k correct to 3 decimal places. [4]

6 Employees at a particular company have been working seven hours each day, from 9 am to 4 pm. To try to reduce absence, the company decides to introduce 'flexi-time' and allow employees to work their seven hours each day at any time between 7 am and 9 pm. For a random sample of 10 employees, the numbers of hours of absence in the year before and the year after the introduction of flexi-time are given in the following table.

Employee	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Before	42	35	96	74	20	5	78	45	146	0
After	34	32	100	72	31	2	61	35	140	0

Use a paired sample *t*-test to test, at the 10% significance level, whether the population mean number of hours of absence has decreased, following the introduction of flexi-time. [7]

7 James throws a discus repeatedly in an attempt to achieve a successful throw. A throw is counted as successful if the distance achieved is over 40 metres. For each throw, the probability that James is successful is $\frac{1}{4}$, independently of all other throws. Find the probability that James takes

(i) exactly 5 throws to achieve the first successful throw, [1]

(ii) more than 8 throws to achieve the first successful throw. [2]

In order to qualify for a competition, a discus-thrower must throw over 40 metres within at most six attempts. When a successful throw is achieved, no further throws are taken. Find the probability that James qualifies for the competition. [2]

Colin is another discus-thrower. For each throw, the probability that he will achieve a throw over 40 metres is $\frac{1}{3}$, independently of all other throws. Find the probability that exactly one of James and Colin qualifies for the competition. [3]

8 A random sample of 200 is taken from the adult population of a town and classified by age-group and preferred type of car. The results are given in the following table.

	Hatchback	Estate	Convertible
Under 25 years	32	11	17
Between 25 and 50 years	45	24	6
Over 50 years	31	16	18

Test, at the 5% significance level, whether preferred type of car is independent of age-group. [8]

9 The continuous random variable X has distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2, \\ \frac{1}{8}x - \frac{1}{4} & 2 \leqslant x \leqslant 10, \\ 1 & x > 10. \end{cases}$$

Find the value of k for which $\mathrm{P}(X \geqslant k) = 0.6$. [3]

The random variable Y is defined by $Y = 2\ln X$. Find the distribution function of Y. [3]

Find the probability density function of Y and sketch its graph. [4]

10 The lengths of a random sample of eight fish of a certain species are measured, in cm, as follows.

17.3 15.8 18.2 15.6 16.0 18.8 15.3 15.0

Assuming that lengths are normally distributed,

(i) test, at the 10% significance level, whether the population mean length of fish of this species is greater than 15.8 cm, [7]

(ii) calculate a 95% confidence interval for the population mean length of fish of this species. [4]

11 Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is suspended from a fixed point by a light elastic string of natural length l, and hangs in equilibrium. The particle is pulled vertically down to a position where the length of the string is $\frac{13}{7}l$. The particle is released from rest in this position and reaches its greatest height when the length of the string is $\frac{11}{7}l$.

(i) Show that the modulus of elasticity of the string is $\frac{7}{5}mg$. [2]

(ii) Show that P moves in simple harmonic motion about the equilibrium position and state the period of the motion. [4]

(iii) Find the time after release when the speed of P is first equal to half of its maximum value. [6]

OR

For a random sample of 12 observations of pairs of values (x, y), the equation of the regression line of y on x and the equation of the regression line of x on y are

$$y = bx + 4.5 \qquad \text{and} \qquad x = ay + c$$

respectively, where a, b and c are constants. The product moment correlation coefficient for the sample is 0.6.

(i) Test, at the 5% significance level, whether there is evidence of positive correlation between the variables. [4]

(ii) Given that $b - a = 0.5$, find the values of a and b. [3]

(iii) Given that the sum of the x-values in the sample data is 66, find the value of c and sketch the two regression lines on the same diagram. [3]

For each of the 12 pairs of values of (x, y) in the sample, another variable z is considered, where $z = 5y$.

(iv) State the coefficient of x in the equation of the regression line of z on x and find the value of the product moment correlation coefficient between x and z, justifying your answer. [2]

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